Teaching Tip: The Limit of (sin*t*)/*t*

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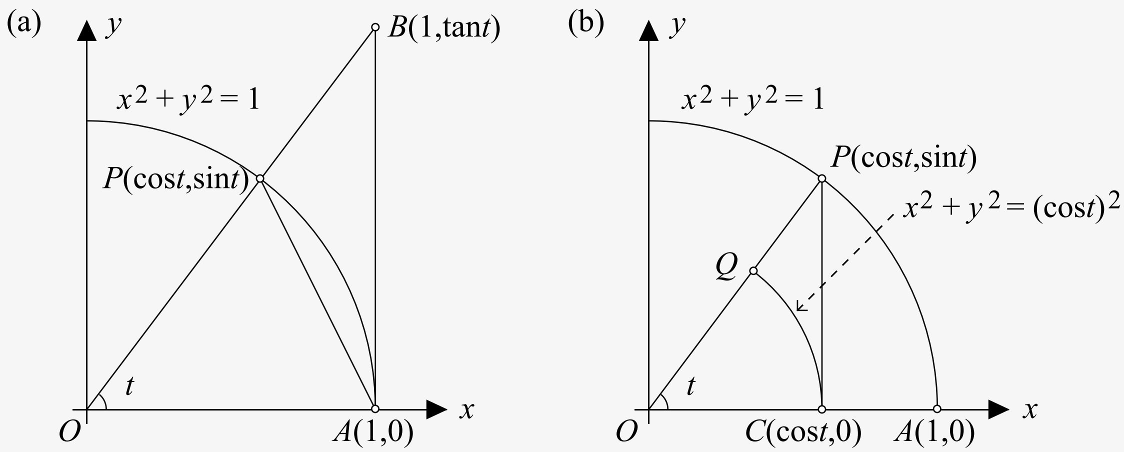
Almost all modern college calculus texts include a proof that $\lim_{t\to 0}(\sin t)/t = 1$. Most do so in the same way: for t in $(0,\pi/2)$ compare the areas of the circular sector and two triangles illustrated in part (a) of the figure. The area of ΔOAP is $\sin t/2$; the area of sector OAP is $t/2$; the area of ΔOAB is $\tan t/2$; and thus $(\sin t)/2 \le t/2 \le (\tan t)/2$. These inequalities are now cleverly manipulated to obtain $\cos t \le (\sin t)/t \le 1$, from which it follows that $\lim_{t\to 0^+}(\sin t)/t = 1$ (the left-hand limit is usually obtained by noting that $\sin(-t)/(-t) = \sin t/t$). In our experience, it is the manipulation of the inequalities that may cause some students difficulty.



The manipulation of the inequalities can be greatly simplified by using *one* triangle and *two* circular sectors, as illustrated in part (b) of the Figure. The area of sector OCQ is $t(\cos t)^2/2$; the area of ΔOCP is $(\sin t\cos t)/2$; the area of sector OAP is $t/2$; and hence $t(\cos t)^2/2 \le (\sin t\cos t)/2 \le t/2$. Multiplication by $2/t\cos t$ immediately yields $\cos t \le (\sin t)/t \le 1/\cos t$, from which the limit follows.